Sheet 1

1. Find the vector $\bar{A}$ directed from (2,-4,1) to (0,-2,0) in Cartesian coordinates and find the unit vector along $\bar{A}$

$$\begin{bmatrix} \bar{A} = -2\bar{a}_x + 2\bar{a}_y - \bar{a}_z \\ \bar{a}_A = \frac{-2}{3}\bar{a}_x + \frac{2}{3}\bar{a}_y - \frac{1}{3}\bar{a}_z \end{bmatrix}$$

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2. Show that $\bar{A} = 4\bar{a}_x - 2\bar{a}_y - \bar{a}_z$ and $\bar{B} = \bar{a}_x + 4\bar{a}_y - 4\bar{a}_z$ are perpendicular

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3. Determine the smaller angle between

$$\bar{A} = 2\bar{a}_x + 4\bar{a}_y \text{ and } \bar{B} = 6\bar{a}_y - 4\bar{a}_z$$

using the cross product and also the dot product

$$\begin{bmatrix} \theta_{AB} = 41.9088^\circ \\ \theta_{AB} = 138.09^\circ \end{bmatrix}$$

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4. Given $\bar{F} = (y-1)\bar{a}_x + 2x\bar{a}_y$, find the vector at (2,2,1) and its projection on $\bar{B} = 5\bar{a}_y - \bar{a}_y + 2\bar{a}_z$

$$\begin{bmatrix} \bar{F}|_{(2,2,1)} = \bar{a}_x + 4\bar{a}_y \\ \text{Projection of } \bar{F} \text{ onto } \bar{B} = \frac{1}{\sqrt{30}} \end{bmatrix}$$

5 If $\bar{A} = \bar{a}_x + 2\bar{a}_y - 3\bar{a}_z$ and $\bar{B} = 2\bar{a}_x - \bar{a}_y + \bar{a}_z$

Determine :

a) The magnitude of projection of $\bar{B}$ on $\bar{A}$

b) The smallest angle between $\bar{A}$ and $\bar{B}$

c) The vector projection $\bar{A}$ onto $\bar{B}$

d) A unit vector perpendicular to the plane containing $\bar{A}$ and $\bar{B}$

$$\left[\begin{array}{c} \textbf{Magnitude Projection of } \bar{\boldsymbol{B}} \textbf{ onto } \bar{\boldsymbol{A}} = \frac{\mathbf{3}}{\sqrt{\mathbf{14}}} \\ \boldsymbol{\theta_{AB}} = \mathbf{109.1066^\circ} \\ \textbf{Vector Projection of } \bar{\boldsymbol{B}} \textbf{ onto } \bar{\boldsymbol{A}} = -\bar{\boldsymbol{a}}_x + \mathbf{0.5}\bar{\boldsymbol{a}}_y - \mathbf{0.5}\bar{\boldsymbol{a}}_z \\ \bar{\boldsymbol{a}}_n = \pm\frac{\bar{\boldsymbol{a}}_x + \mathbf{7}\bar{\boldsymbol{a}}_y + \mathbf{5}\bar{\boldsymbol{a}}_z}{\sqrt{\mathbf{75}}} \end{array}\right]$$

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6 Given $\bar{A} = \bar{a}_x + \bar{a}_y$, $\bar{B} = \bar{a}_x + 2\bar{a}_z$, $\bar{C} = 2\bar{a}_y + \bar{a}_z$

Find $(\bar{A} \times \bar{B}) \times \bar{C}$ and compare it with $\bar{A} \times (\bar{B} \times \bar{C})$, comment on the result.

$$\left[\begin{array}{c} (\bar{\boldsymbol{A}} \times \bar{\boldsymbol{B}}) \times \bar{\boldsymbol{C}} = -\mathbf{2}\bar{\boldsymbol{a}}_y + \mathbf{4}\bar{\boldsymbol{a}}_z \\ \bar{\boldsymbol{A}} \times (\bar{\boldsymbol{B}} \times \bar{\boldsymbol{C}}) = \mathbf{2}\bar{\boldsymbol{a}}_x - \mathbf{2}\bar{\boldsymbol{a}}_y + \mathbf{3}\bar{\boldsymbol{a}}_z \end{array}\right]$$

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7 Find $\bar{A} \cdot \bar{B} \times \bar{C}$ for $\bar{A}$, $\bar{B}$, $\bar{C}$ of problem 6 and compare it with $\bar{A} \times \bar{B} \cdot \bar{C}$, comment on the result

$$[\bar{\boldsymbol{A}} \cdot \bar{\boldsymbol{B}} \times \bar{\boldsymbol{C}} = \bar{\boldsymbol{A}} \times \bar{\boldsymbol{B}} \cdot \bar{\boldsymbol{C}} = -\mathbf{5}]$$

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8 Express the unit vector which is directed toward the origin from an arbitrary point on the plane $z = -5$

$$\left[\bar{\boldsymbol{a}}_R = \frac{-\boldsymbol{x}\bar{\boldsymbol{a}}_x - \boldsymbol{y}\bar{\boldsymbol{a}}_y + \mathbf{5}\bar{\boldsymbol{a}}_z}{\sqrt{\boldsymbol{x}^2 + \boldsymbol{y}^2 + \mathbf{25}}}\right]$$

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9 Given the two vectors $\bar{A} = -\bar{a}_x - 3\bar{a}_y - 4\bar{a}_z$, $\bar{B} = 2\bar{a}_x + 2\bar{a}_y + 2\bar{a}_z$ and a point $C(1,3,4)$, Find

(a) R_{AB} (b) $|\bar{A}|$ (c) $\bar{a}_A$ (d) $\bar{a}_{AB}$

(e) a unit vector directed from C toward A

$$\begin{bmatrix} \bar{\boldsymbol{R}}_{AB} = \mathbf{3}\bar{\boldsymbol{a}}_x + \mathbf{5}\bar{\boldsymbol{a}}_y + \mathbf{6}\bar{\boldsymbol{a}}_z \\ |\bar{\boldsymbol{A}}| = \sqrt{\mathbf{26}} \\ \bar{\boldsymbol{a}}_A = \dfrac{-\bar{\boldsymbol{a}}_x - \mathbf{3}\bar{\boldsymbol{a}}_y - \mathbf{4}\bar{\boldsymbol{a}}_z}{\sqrt{\mathbf{26}}} \\ \bar{\boldsymbol{a}}_{AB} = \dfrac{\mathbf{3}\bar{\boldsymbol{a}}_x + \mathbf{5}\bar{\boldsymbol{a}}_y + \mathbf{6}\bar{\boldsymbol{a}}_z}{\sqrt{\mathbf{70}}} \\ \bar{\boldsymbol{a}}_{CA} = \dfrac{-\bar{\boldsymbol{a}}_x - \mathbf{3}\bar{\boldsymbol{a}}_y - \mathbf{4}\bar{\boldsymbol{a}}_z}{\sqrt{\mathbf{26}}} \end{bmatrix}$$

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10 A triangle is defined by three points $A\ (2,-5,1) B\ (-3,2,4) C\ (0,3,1)$ Find

a) $R_{BC} \times R_{BA}$

b) The area of the triangle

c) A unit vector perpendicular to the plane of the triangle

$$\begin{bmatrix} \bar{\boldsymbol{R}}_{BC} \times \bar{\boldsymbol{R}}_{BA} = -\mathbf{24}\bar{\boldsymbol{a}}_x - \mathbf{6}\bar{\boldsymbol{a}}_y - \mathbf{26}\bar{\boldsymbol{a}}_z \\ \text{Area} = \sqrt{\mathbf{322}} = \mathbf{17.94436}\ \textbf{square unit} \\ \bar{\boldsymbol{a}}_n = \pm\dfrac{\mathbf{12}\bar{\boldsymbol{a}}_x + \mathbf{3}\bar{\boldsymbol{a}}_y + \mathbf{13}\bar{\boldsymbol{a}}_z}{\sqrt{\mathbf{322}}} \end{bmatrix}$$